

## Decomposition of $N(x)/D(x)$ into Partial Fractions

1. Divide if improper: If  $N(x)/D(x)$  is an improper fraction (that is, if the degree of the numerator is greater than or equal to the degree of the denominator), divide the denominator into the numerator to obtain

$$\frac{N(x)}{D(x)} = (\text{a polynomial}) + \frac{N_1(x)}{D(x)}$$

where the degree of  $N_1(x)$  is less than the degree of  $D(x)$ . Then apply steps 2, 3, and 4 to the proper rational expression  $N_1(x)/D(x)$ .

2. Factor denominator: Completely factor the denominator into factors of the form

$$(px + q)^m \qquad \text{and} \qquad (ax^2 + bx + c)^n$$

where  $ax^2 + bx + c$  is irreducible.

3. Linear Factors: For each factor of the form  $(px + q)^m$ , the partial fraction decomposition must include the following sum of  $m$  fractions.

$$\frac{A_1}{(px + q)} + \frac{A_2}{(px + q)^2} + \cdots + \frac{A_m}{(px + q)^m}$$

4. Quadratic Factors: For each factor of the form  $(ax^2 + bx + c)^n$ , the partial fraction decomposition must include the following sum of  $n$  fractions.

$$\frac{B_1x + C_1}{(ax^2 + bx + c)} + \frac{B_2x + C_2}{(ax^2 + bx + c)^2} + \cdots + \frac{B_nx + C_n}{(ax^2 + bx + c)^n}$$