Trigonometric Substitutions

Substitutions

- $u = a\tan\theta$ replaces $a^2 + u^2$

  requires $\theta = \tan^{-1}(\frac{u}{a})$ with $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

- $u = a\sin\theta$ replaces $a^2 - u^2$

  requires $\theta = \sin^{-1}(\frac{u}{a})$ with $-\frac{\pi}{2} \quad \theta \quad \frac{\pi}{2}$

- $u = a\sec\theta$ replaces $u^2 - a^2$

  requires $\theta = \sec^{-1}(\frac{u}{a})$ with $\begin{cases} 0 \quad \theta < \frac{\pi}{2} & \text{if } \frac{u}{a} \geq 1 \\ \frac{\pi}{2} < \theta \quad \pi & \text{if } \frac{u}{a} \quad -1 \end{cases}$

Integrals of Trigonometric Functions

$$\int \sin\theta\, d\theta = -\cos\theta + C$$

$$\int \cos\theta\, d\theta = \sin\theta + C$$

$$\int \tan\theta\, d\theta = -\ln|\cos\theta| + C$$

$$= \ln|\sec\theta| + C$$

$$\int \cot\theta\, d\theta = \ln|\sin\theta| + C$$

$$= -\ln|\csc\theta| + C$$

$$\int \sec\theta\, d\theta = \ln|\sec\theta + \tan\theta| + C$$

$$\int \csc\theta\, d\theta = -\ln|\csc\theta + \cot\theta| + C$$

Useful Trigonometry

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

$$\sin^2\theta = \frac{1 - \cos 2\theta}{2}$$

$$\cos^2\theta = \frac{1 + \cos 2\theta}{2}$$

$$\tan^2\theta = \frac{1 - \cos 2\theta}{1 + \cos 2\theta}$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$= 2\cos^2\theta - 1$$

$$= 1 - 2\sin^2\theta$$

$$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$$